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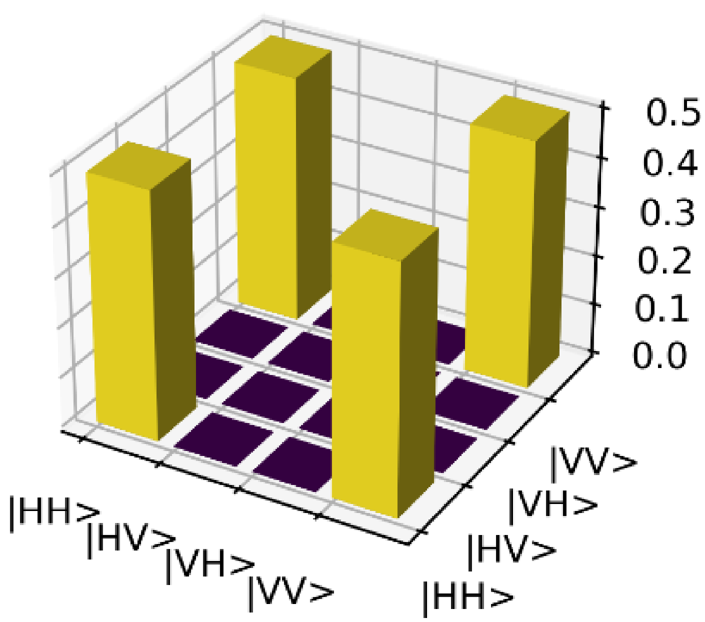
Experimental Quantum Technologies

CHSH Inequality



Introduction and theory

Of particular interest for many emerging implementations of quantum technologies, such as quantum computing or quantum cryptography, are two-level quantum systems. These so-called “quantum bits” or “qubits” [1], named in analogy to a classical “bit”, can be realized using different physical systems such as electronic spins, energy levels of trapped ions or the polarization of photons. Unlike classical bits, however, multiple quantum bits can be prepared in a quantum entangled state, giving rise to properties that cannot be explained with “local hidden variable theories”. In this experiment we will work with photonic qubits, photons that are entangled in their polarization degree of freedom. Using this physical system, we will demonstrate the violation of the so-called Bell inequality [2]. This inequality derived by J. S. Bell in 1964, proposed a test that allowed to accept or refute the postulates of Einstein, Podolsky and Rosen (EPR), widely known as the EPR Paradox [3]. For the actual experiment, we will rely on the CHSH test criterion [4], which translates Bell’s inequality into a more experimentally accessible setting.

Before we go on with the EPR-paradox and Bell’s inequality, let’s add a brief reminder about the mathematical representation of quantum states. Generally, the state of a physical system is defined by a vector in a Hilbert space H. When considering a single qubit, in our case a photon and its polarization, the corresponding Hilbert space has a dimension of $n = 2$, such that each polarization can be expressed as the superposition of two orthogonal basis vectors, e.g. $|H\rangle$ and $|V\rangle$. Using this basis of horizontal (H) and vertical (V) polarization, a general state $|\psi\rangle$ of a single polarization qubit can be written as

$$|\psi\rangle = a|H\rangle + b|V\rangle \qquad (1)$$

with the normalization condition for the complex amplitudes a, b: $|a|^2 + |b|^2 = 1$.

In a next step we want to consider now two qubits, i.e. two photons, which are possibly entangled in their polarization degree of freedom. The corresponding Hilbert space for the two-qubit case is obtained from the tensor product of the two single-qubit spaces $H_{1,2}$: $H = H_1 \otimes H_2$. As a result, we have to consider a four-dimensional Hilbert space now, where we obtain the basis vectors from tensor products of the single qubit space:

$$\begin{aligned} |HH\rangle &= |H\rangle_1 \otimes |H\rangle_2, \qquad |HV\rangle = |H\rangle_1 \otimes |V\rangle_2, \\ |VV\rangle &= |V\rangle_1 \otimes |V\rangle_2, \qquad |VH\rangle = |V\rangle_1 \otimes |H\rangle_2. \end{aligned} \qquad (2)$$

With this, a general two-qubit state can be written as

$$|\psi\rangle = a_{HH}|HH\rangle + a_{HV}|HV\rangle + a_{VH}|VH\rangle + a_{VV}|VV\rangle. \qquad (3)$$

We can now distinguish two important cases:

1. The two-qubit state can be directly written as a (tensor-) product of single-qubit states, e.g.

   $$|\psi\rangle_{sep} = |H\rangle_1 \otimes |H\rangle_2 = |HH\rangle. \qquad (4)$$

   These types of states are referred to as “separable” or “product” states. In this case no entanglement exists between the two qubits.

2. The two-qubit state cannot be factorized into a simple product of single qubit states, an example being

$$|\psi\rangle_{ent} = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle). \quad (5)$$

   In such a case, the state is said to be "entangled".

The at first sight somewhat formal difference between the separable and entangled state actually has far-reaching consequences. Most importantly, in the case of a separable state, the measurement on one photon cannot influence the second photon, if both are spatially separated. This means both photons can be described locally. In contrast, for an entangled state, a measurement on one photon of the pair will always affect the outcome of a measurement performed on the second photon, even if they are spatially separated.

A method to test the non-locality of such a system was developed by John Stewart Bell in 1964. In our experiment, the investigated physical system will consist of two photonic qubits, which are prepared in the so-called Bell states, which by convention are named as:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle), \quad (6)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|HH\rangle - |VV\rangle), \quad (7)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle), \quad (8)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle). \quad (9)$$

Since the initial proposal from Bell is experimentally hard to implement, in this lab course we will measure a different inequality, proposed by John Clauser, Michael Horne, Abner Shimony and Richard Holt (CHSH) five years later [4]. The derivation of the inequality can be found in [1] and only the result

$$S(\alpha, \alpha', \beta, \beta') = |E(\alpha, \beta) + E(\alpha', \beta) - E(\alpha, \beta') + E(\alpha', \beta')| \leq 2. \quad (10)$$

will be used here.

The test parameter $S(\alpha, \alpha', \beta, \beta')$ is less than or equal to 2 if the examined physical system admits a local description. $E(\alpha, \beta)$ is the normalized expectation value of a correlation measurement, i.e. a simultaneous measurement on each separate qubit of the entangled state at different locations. In our case these measurements will be projections onto a particular polarization basis state, which practically means that both photons are sent through two separate polarizers and afterwards detected by a single photon detector as show in fig. 1.

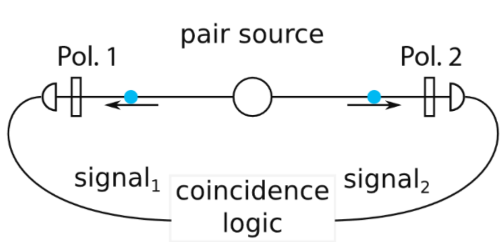


Figure 1: Coincident measurement of two photons passing through different polarizers.

References

[1] J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, "Proposed Experiment to Test Local Hidden-Variable Theories," *Physical Review Letters,* vol. 23, no. 15, pp. 880-884, 1969.