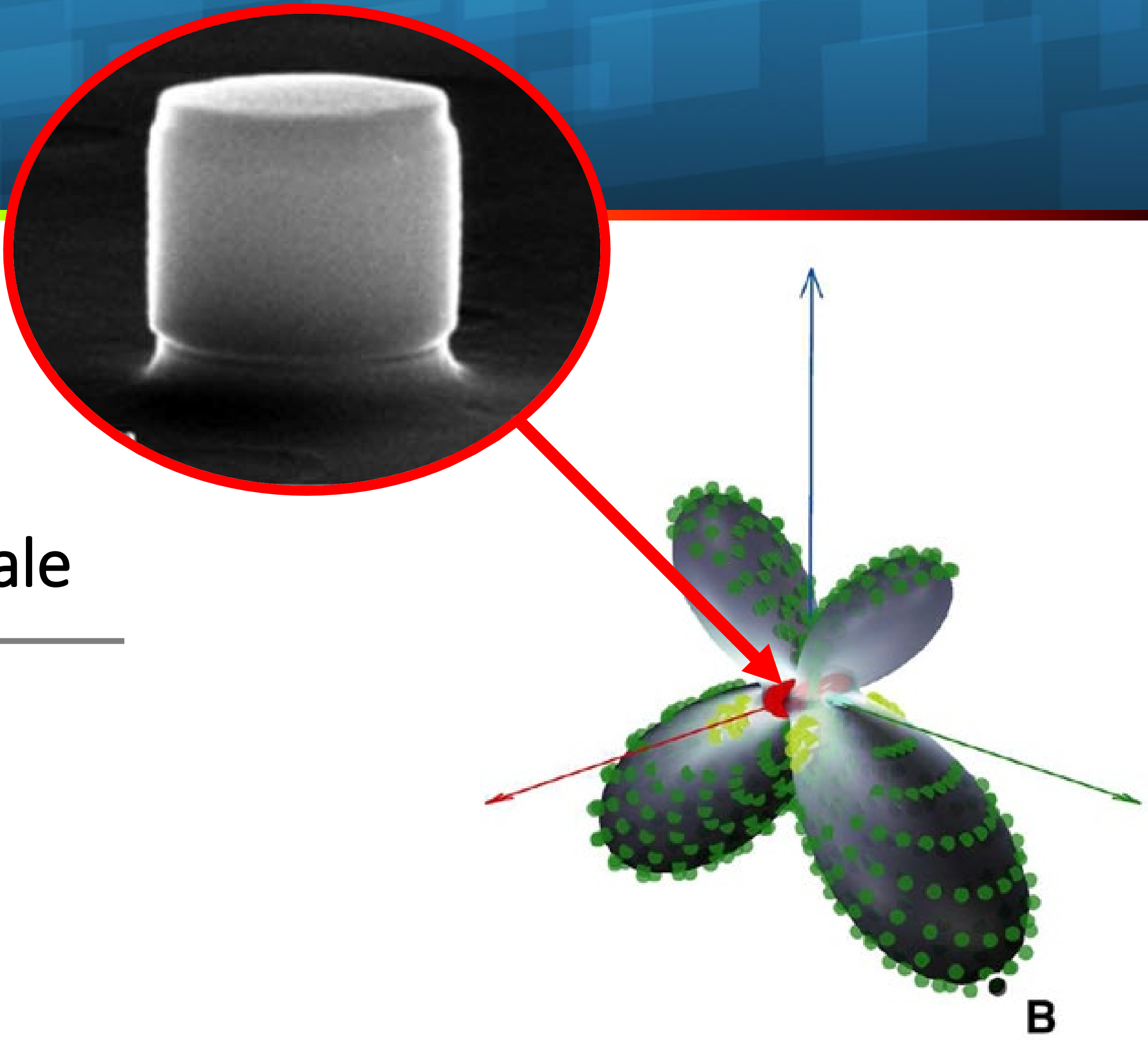
Institute of
Applied Physics
Friedrich-Schiller-Universität Jena



Entanglement generation at the nanoscale

Frank Setzpfandt

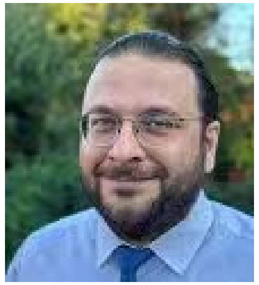
Acknowledgements

FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Université de Paris

Maximilian Weissflog

Sina Saravi



Thomas Pertsch

Romain Dezert

Adrien Borne

Giuseppe Leo

Vincent Vinel

Carlo Gigli

DFG Deutsche Forschungsgemeinschaft

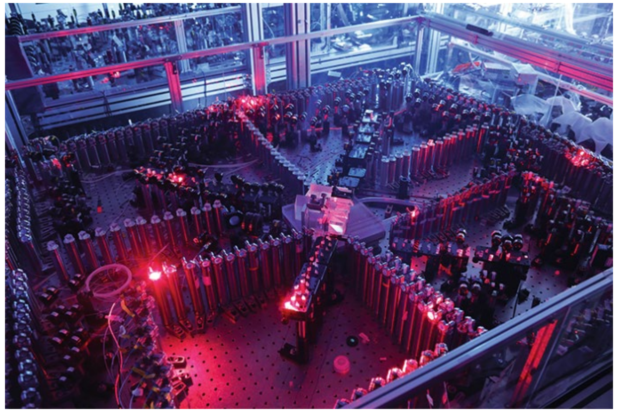
META ACTIVE

NOA NONLINEAR OPTICS DOWN TO ATOMIC SCALES

Bundesministerium für Bildung und Forschung

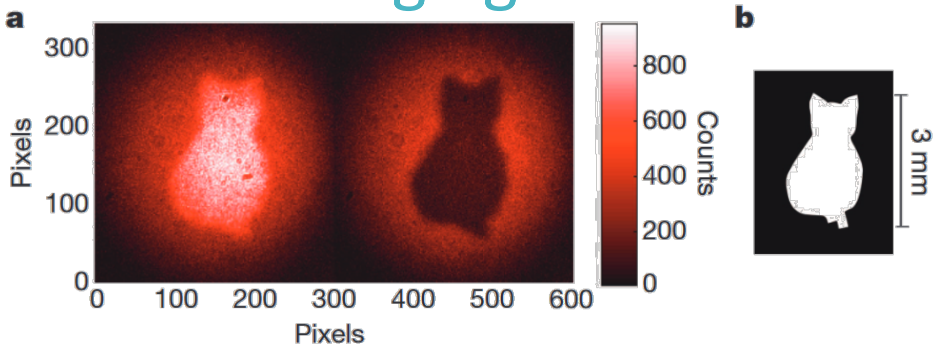
Quantum Optics… Nice, but what for?

Quantum-Computing



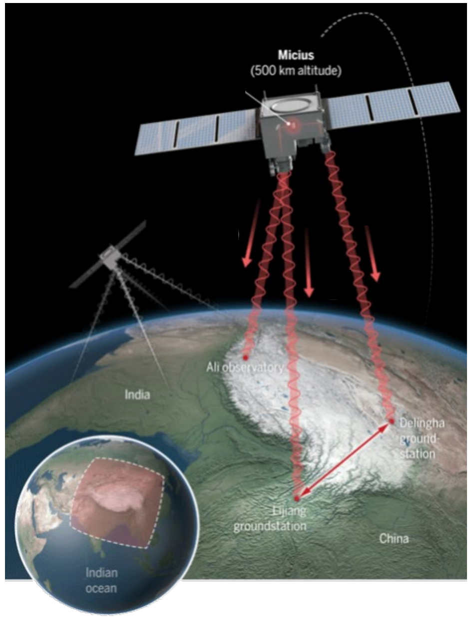
H.-S. Zhong *et al.*, *Science* **370**, 1460 (2020)

Quantum-Imaging



G. B. Lemos, et. al., *Nature* **512**, 409 (2014).

Quantum-Communication



J. Yin *et al.*, *Science* **356**, 1140 (2017)

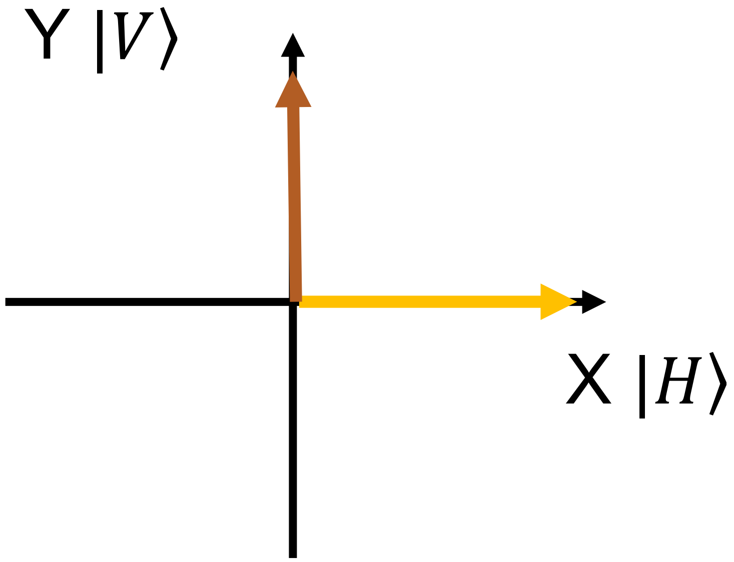
Needs entangled photons

What is (polarization) entanglement…

Non-entangled (separable)
2-Photon quantum state:

$$|\psi\rangle = |H_1 V_2\rangle$$

Polarization basis



Important: we need to know what is “1” and “2”

Let’s define a “signal” and “idler” mode e.g.:

- Different frequencies
- Different directions

What is (polarization) entanglement…

Entangled 2-Photon quantum state:

$$|\psi\rangle = |H_1 V_2\rangle \pm |V_1 H_2\rangle$$

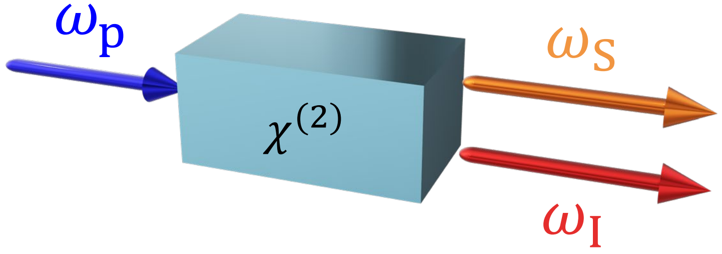
or

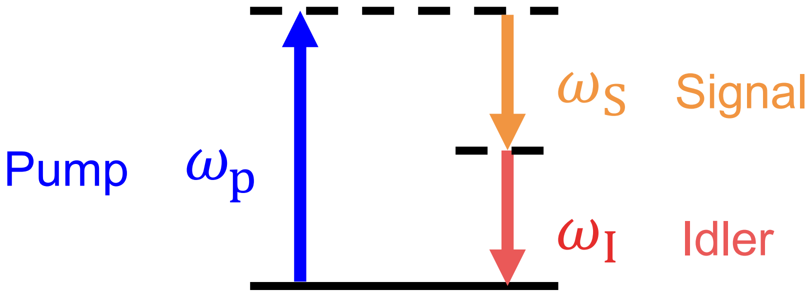
$$|\psi\rangle = |H_1 H_2\rangle \pm |V_1 V_2\rangle$$

Coherent super position of both possibilities.

How do we make that practically?

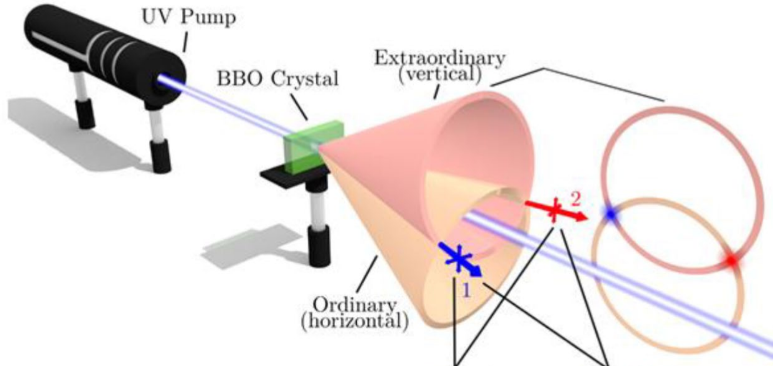
We use “**S**pontaneous **P**arametric **D**own **C**onversion” (SPDC)





Implementations of entangled SPDC sources

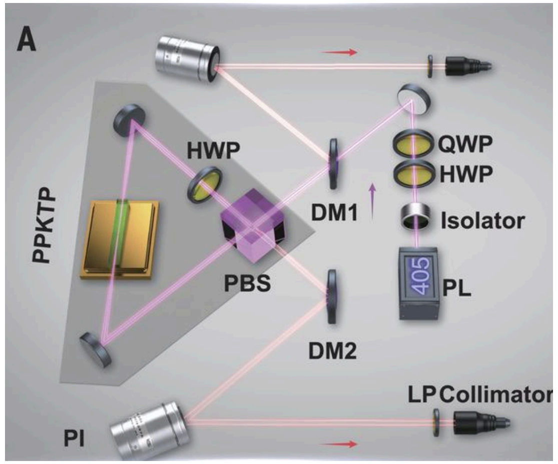
Spatial Post-Selection



$$|\psi\rangle = |H_1 V_2\rangle + e^{i\phi}|V_1 H_2\rangle$$

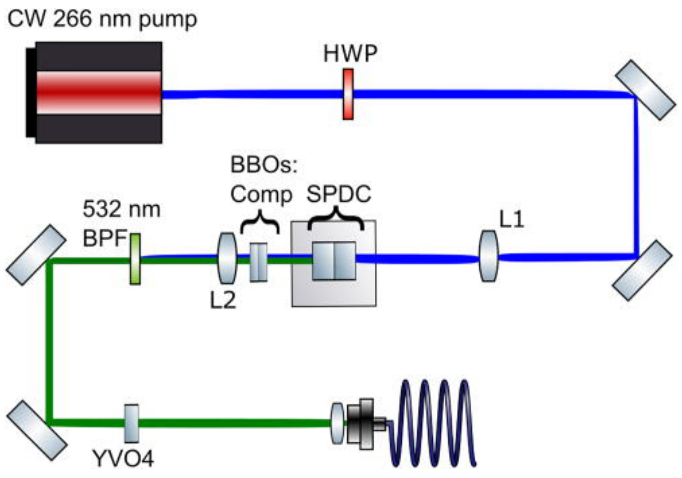
C. Couteau, *Contemporary Physics*, **59**, 3, pp. 291–304, (2018)

Sagnac-Interferometers



J. Yin *et. al.*, *Science*, (2017)

Two crossed crystals

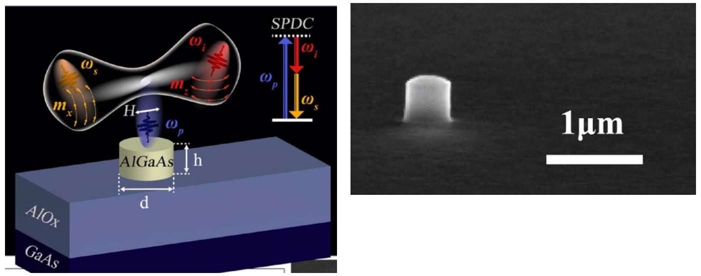


A. S. Perna, et. al, Appl. Phys. Lett. 120, 074001 (2022)

What about nanostructures?

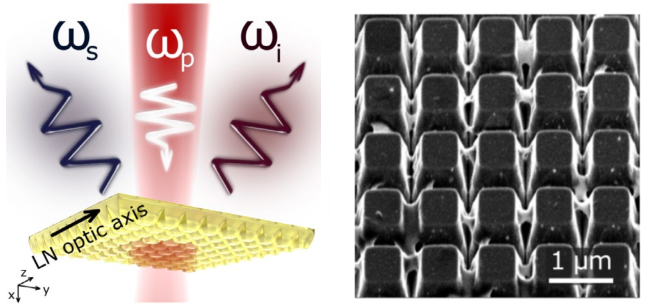
SPDC and entanglement generation in nanostructures

Single (100)-AlGaAs Nanocylinders



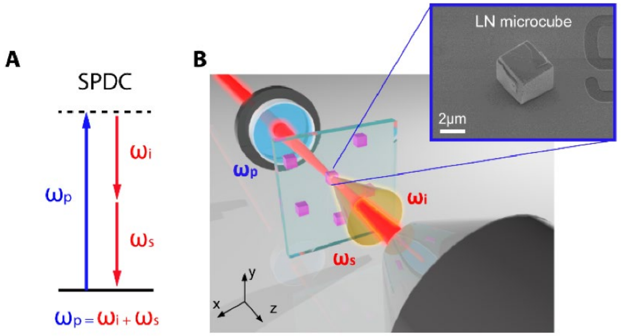
G. Marino *et al.*, *Optica*, **6**, 1416, (2019)

LN-Metasurface



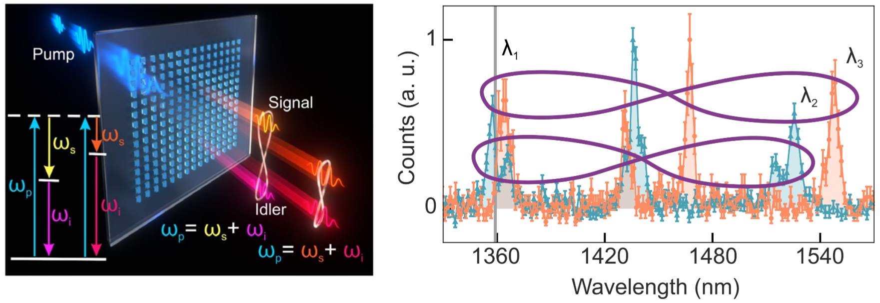
T. Santiago-Cruz *et. al., Nano Lett.*, **21**, 4423–4429 (2021)

Single LN Microcubes



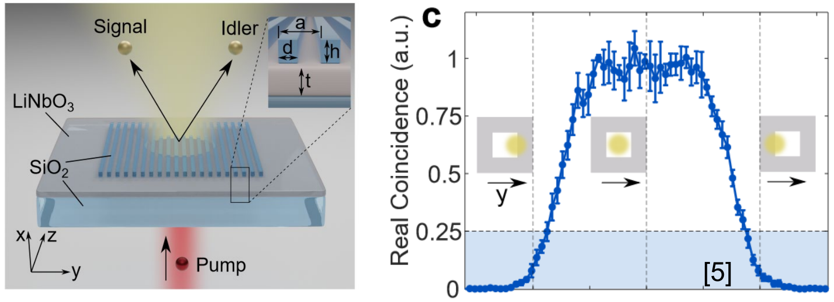
N. M. H. Duong *et al.*, *Opt. Mater. Express,* **12**, 3696–3704, (2022)

BIC resonant GaAs metasurface



T. Santiago-Cruz *et al.*, *Science*, **377**, 991–995, (2022)

LN-film + SiO2 grating



J. Zhang, et al. ,Science Advances **8**, eabq4240 (2022)

Modelling SPDC in a Nanoresonator

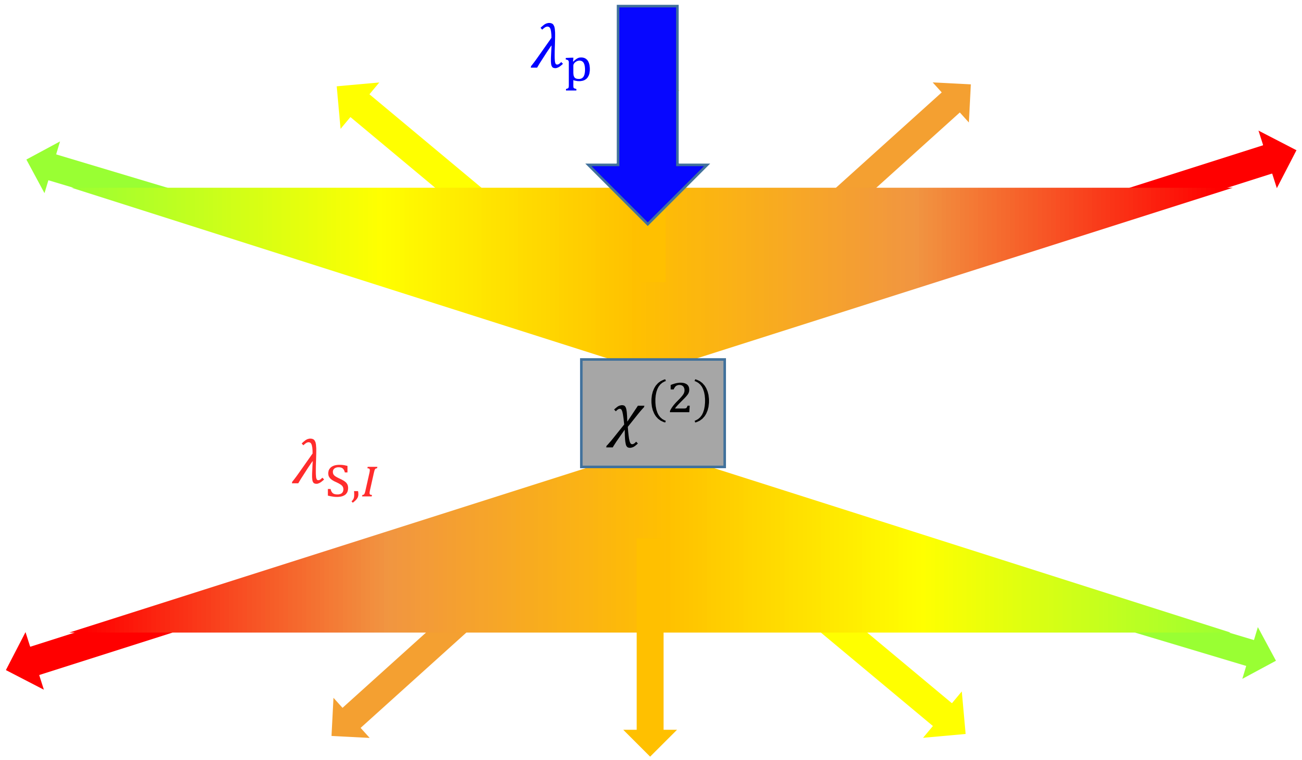
Challenge: Nanoresonators are (typically)

- highly multimodal
- non-Hermitian (open)

Solution: Green's function formalism[1]:

Not relying on quantization of classical modes

→ Handles lossy/radiative systems



[1]A. Poddubny, *et al.* *PRL* **117**, 123901, (2016).

Computing the SPDC Wavefunction for Photonic Nanostructures

Green's function formalism:[1]

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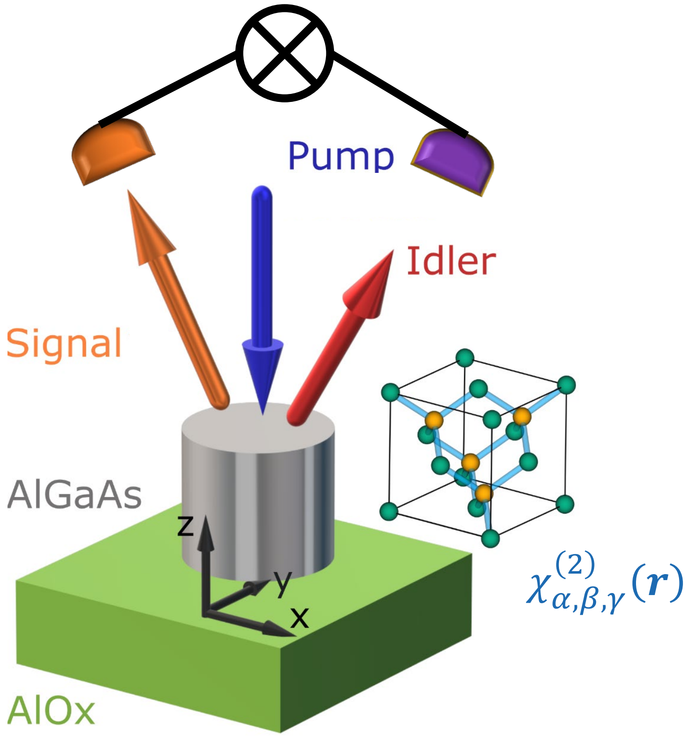
SPDC wavefunction ("two-photon transition probability") $T_{s,i}$:[1]

$$\tilde{T}_{s,i}(\boldsymbol{r}_s, \omega_s, \boldsymbol{e}_s; \boldsymbol{r}_i, \omega_i, \boldsymbol{e}_i) = \sum_{\alpha,\beta,\gamma,q_s,q_i} e_{s,q_s} e_{i,q_i} \int d^3\boldsymbol{r}\, \chi^{(2)}_{\alpha,\beta,\gamma}(\boldsymbol{r}) E_{p,\gamma}(\boldsymbol{r}, \omega_s + \omega_i) G_{q_s,\alpha}(\boldsymbol{r}_s, \boldsymbol{r}, \omega_s) G_{q_i,\beta}(\boldsymbol{r}_i, \boldsymbol{r}, \omega_i)$$

$|\tilde{T}_{s,i}|^2 \propto$ Coincidence rate



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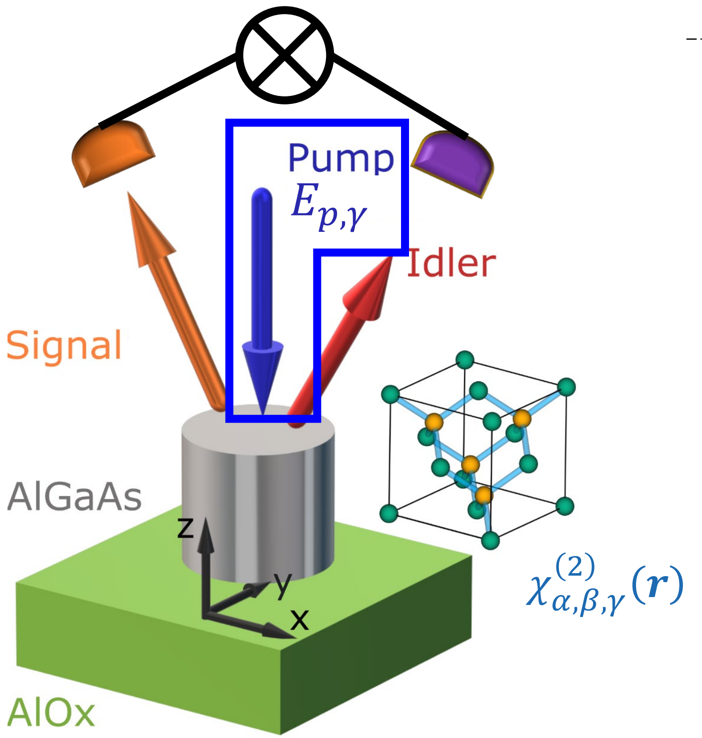
$E_{p,\gamma}(\boldsymbol{r}, \omega_s + \omega_i)$: (Classical) pump field



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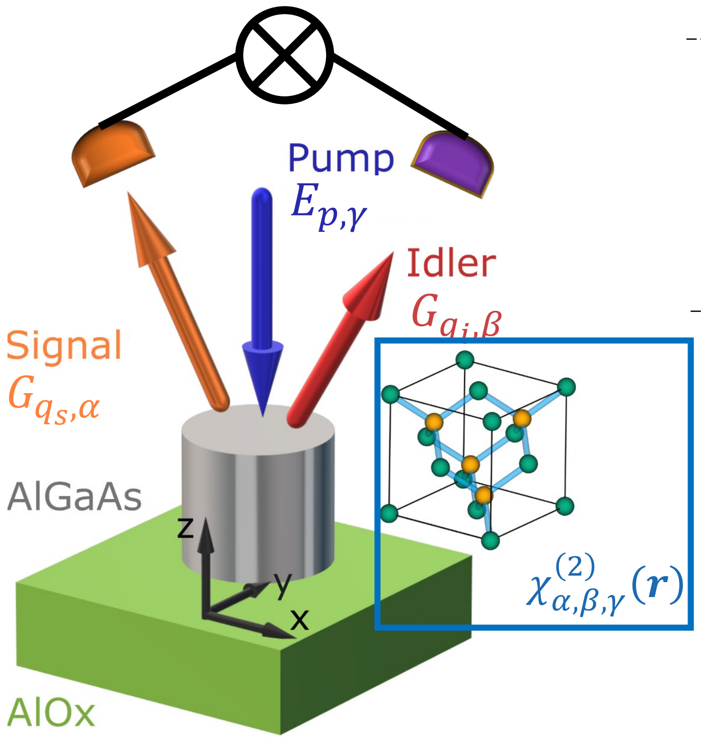

$E_{p,\gamma}(\boldsymbol{r}, \omega_s + \omega_i)$: (Classical) pump field

$\chi^{(2)}_{\alpha,\beta,\gamma}(\boldsymbol{r})$: Material nonlinearity based on crystal structure

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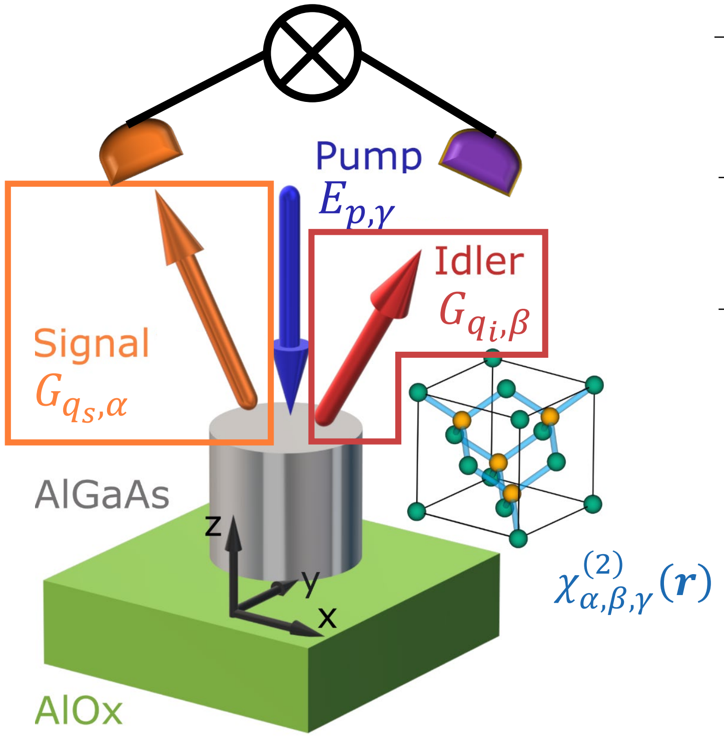

$G_{q_s,\alpha}(\boldsymbol{r}_s, \boldsymbol{r}, \omega_s)$: Green's function at signal frequency

$E_{p,\gamma}(\boldsymbol{r}, \omega_s + \omega_i)$: (Classical) pump field

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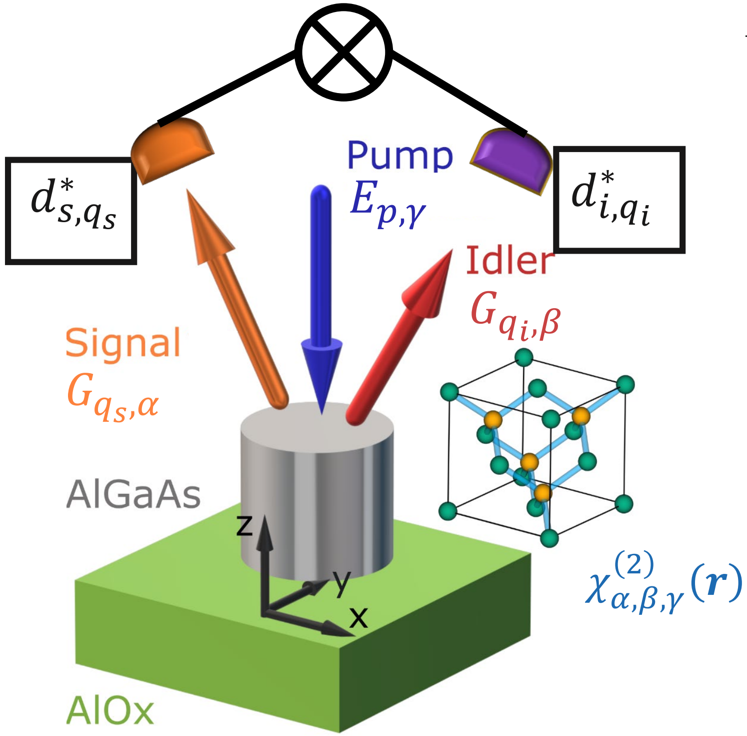
$|T_{s,i}|^2 \propto$ Coincidence rate

d^*_{s,q_s}: Signal detector (2-level system)

$G_{q_s,\alpha}(\boldsymbol{r}_s, \boldsymbol{r}, \omega_s)$: Green's function at signal frequency



$E_{p,\gamma}(\boldsymbol{r}, \omega_s + \omega_i)$: (Classical) pump field

d^*_{i,q_i}: Idler detector (2-level system)

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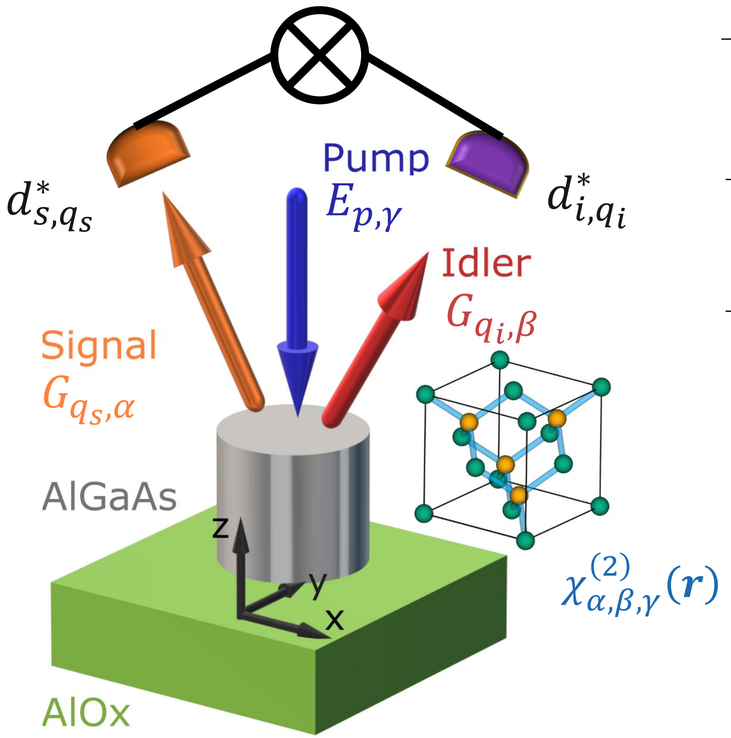

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Green's function is key-component

[1]A. Poddubny, *et al. PRL* **117**, 123901, (2016)

Reconstructing the Green's function

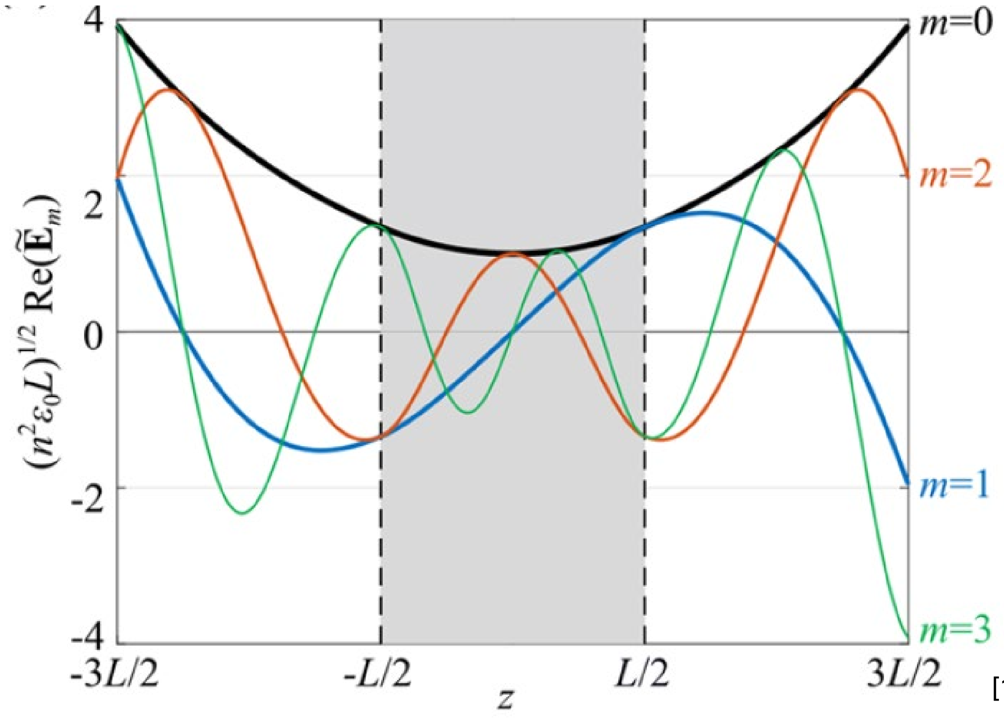
Expansion into eigenmodes of system

$$G_{\alpha,\beta}(\boldsymbol{r},\boldsymbol{r}';\omega) = -\sum_{m=1}^{\infty} \frac{\tilde{E}_{m,\alpha}(\boldsymbol{r}) \times \tilde{E}_{m,\beta}(\boldsymbol{r}')}{(\omega - \tilde{\omega}_m)\tilde{\omega}_m}$$

For a non-Hermitian system the eigenmodes are Quasi-Normal Modes[1,2]

- Also known as "leaky modes", "decaying states"
- Not power-orthogonal ("only quasi-normal")
- Exponentially divergent outside cavity (Normalization!)[1,2]

Simple Photonics Example:
low-Q 1D-Fabry Perot Resonator

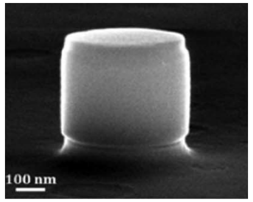


[1]

[1]P. Lalanne, *et al.*, *Laser Photonics Rev.*, **12**, 1700113, (2018)
[2]W. Yan, *et al.*, *PR B*, **97**, 205422 (2018)

Example: AlGaAs Nanocylinder

Reconstruction of Green's function using Quasinormal modes[1,2]:

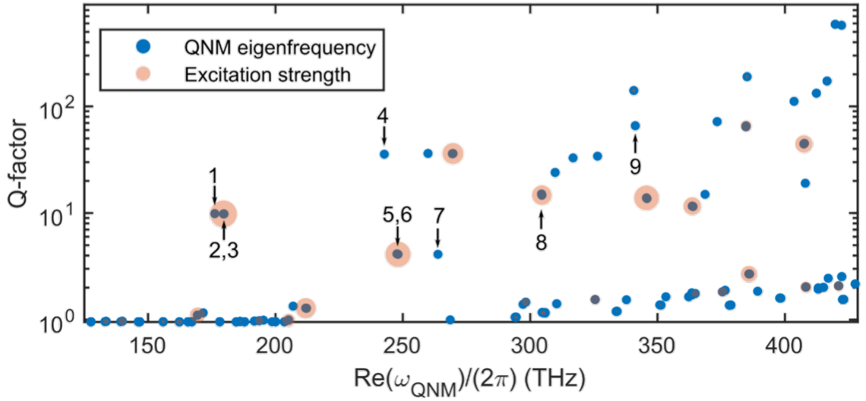


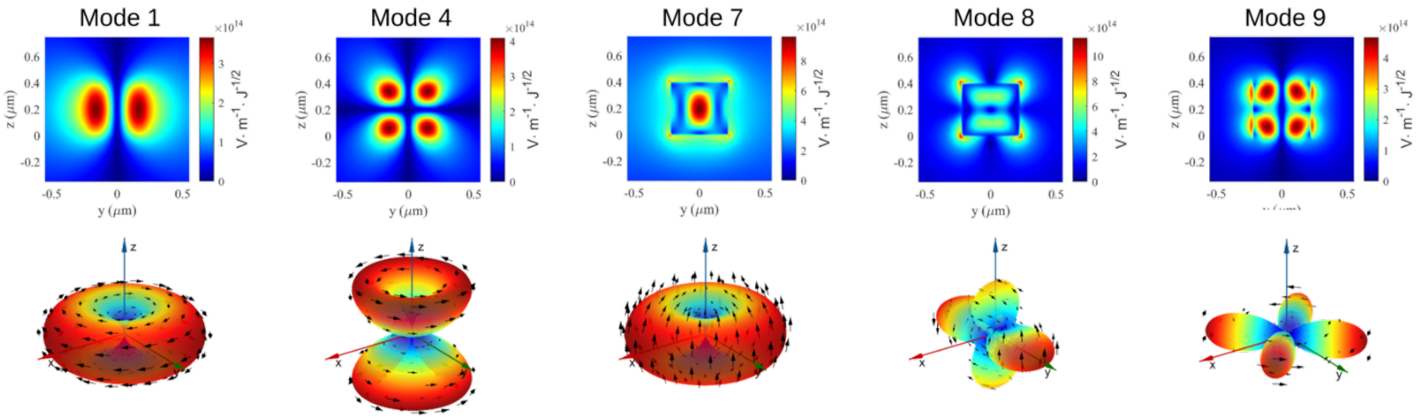
(100)-AlGaAs
h=400 nm, r=200 nm [3]

Maxwell equations formulated as eigenvalue problem

$$\begin{bmatrix} 0 & i\boldsymbol{\epsilon}^{-1}(\boldsymbol{r},\tilde{\omega}_m)\nabla\times \\ -i\boldsymbol{\mu}^{-1}(\boldsymbol{r},\tilde{\omega}_m)\nabla\times & 0 \end{bmatrix}\begin{bmatrix} \tilde{\boldsymbol{E}}_m(\boldsymbol{r}) \\ \tilde{\boldsymbol{H}}_m(\boldsymbol{r}) \end{bmatrix} = \tilde{\omega}_m \begin{bmatrix} \tilde{\boldsymbol{E}}_m(\boldsymbol{r}) \\ \tilde{\boldsymbol{H}}_m(\boldsymbol{r}) \end{bmatrix}$$

Quasi-Normal Modes (QNMs)



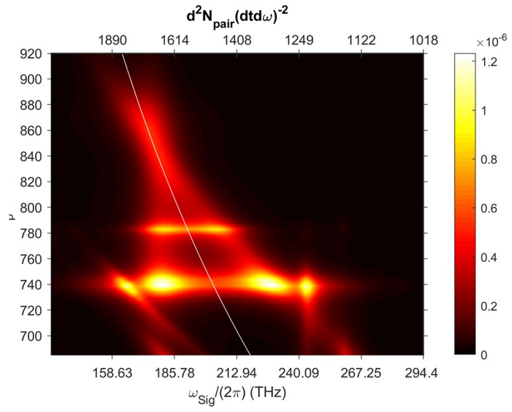


[1]P. Lalanne, *et al.*, *Laser Photonics Rev.*, **12**, 1700113, (2018)
[2]W. Yan, *et al.*, *PR B*, **97,** 205422 (2018)
[3]C. Gigli *et al.*, *JOSA B*, **36**, E55, (2019).

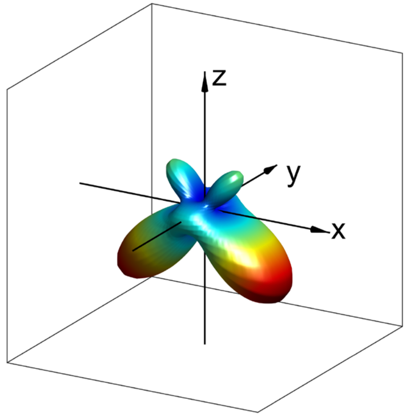
Capabilities of used method

Prediction of SPDC …

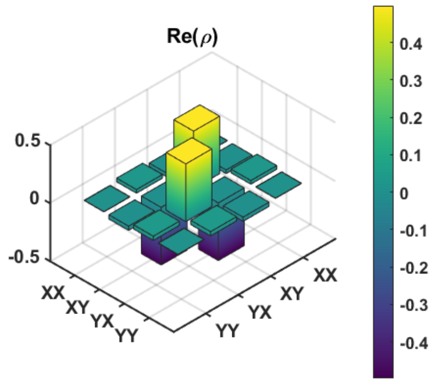
… Spectrum…



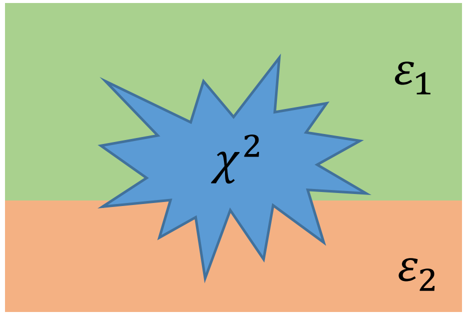
… Emission Direction…



… Polarization State…

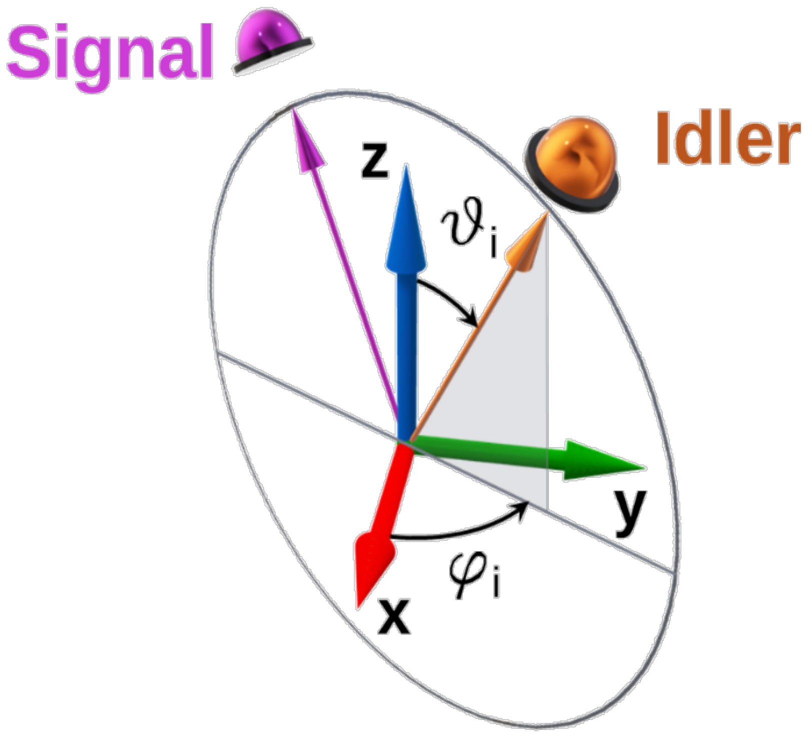


… for **arbitrary nanoresonator shape**
& inhomogeneous background
& dispersive materials.

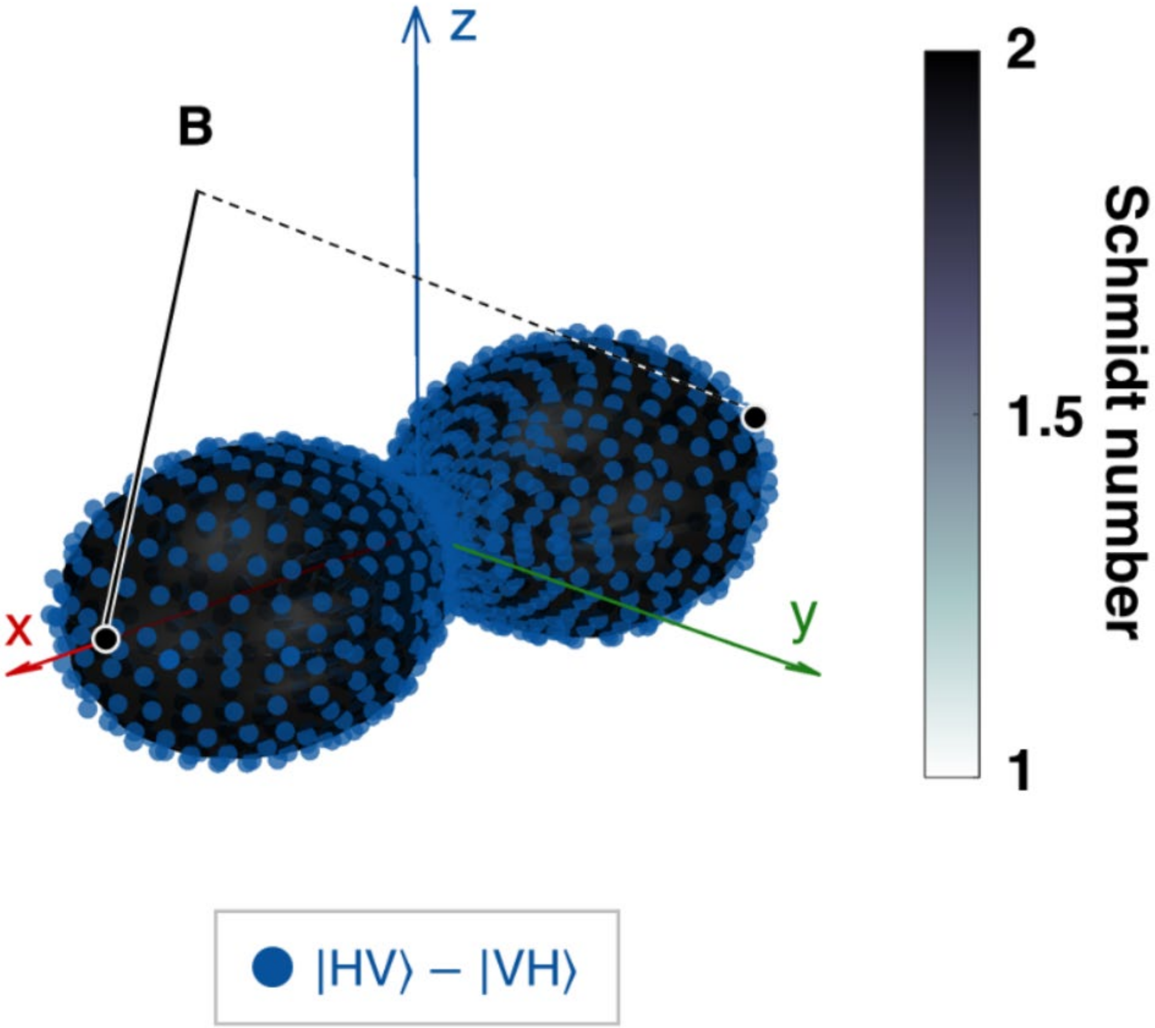


Polarization Entanglement in AlGas Nanocylinder

Measurement geometry



Emitted photon pairs

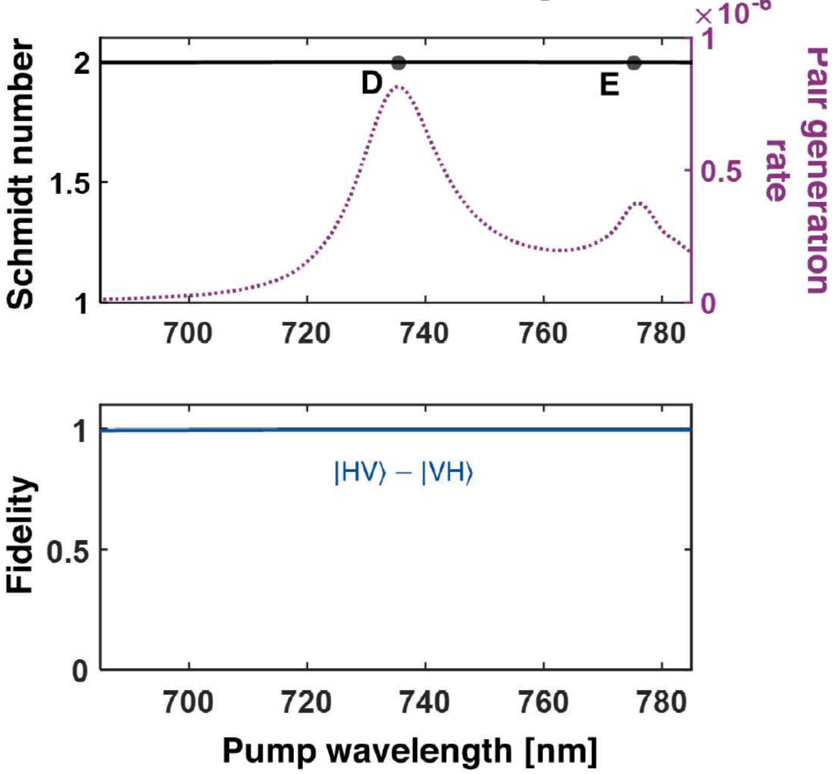


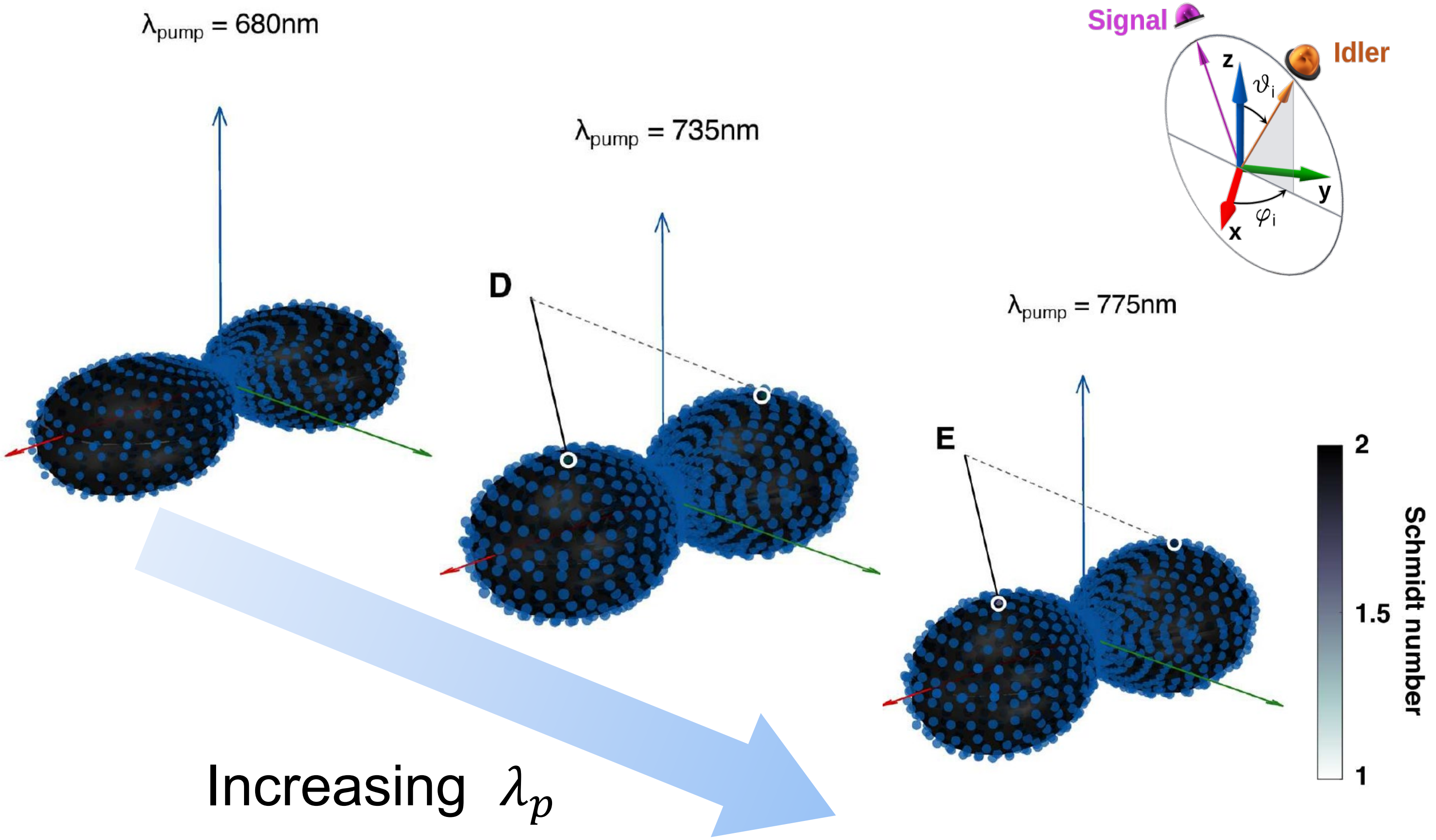
[1]**M. A. Weissflog** *et al.*, „Nonlinear Nanoresonators are Natural Sources of Bell States," Appl. Phys. Rev. 11, 011403 (2024)

Protected Quantum State Generation

Protected generation of entangled quantum state for $\boldsymbol{\varphi}$**-symmetric detection** detection

Change λ_p





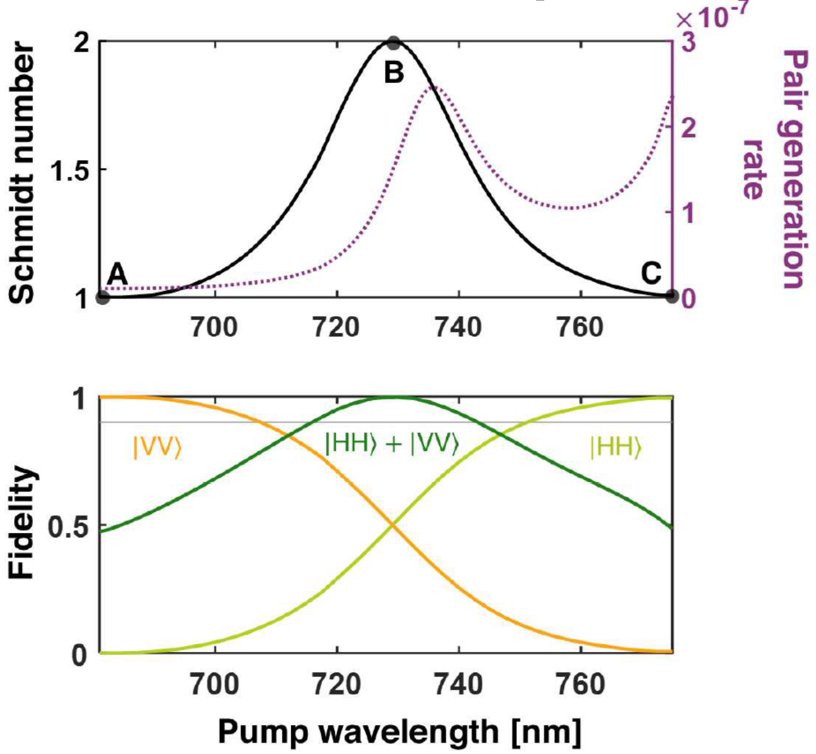
Increasing λ_p

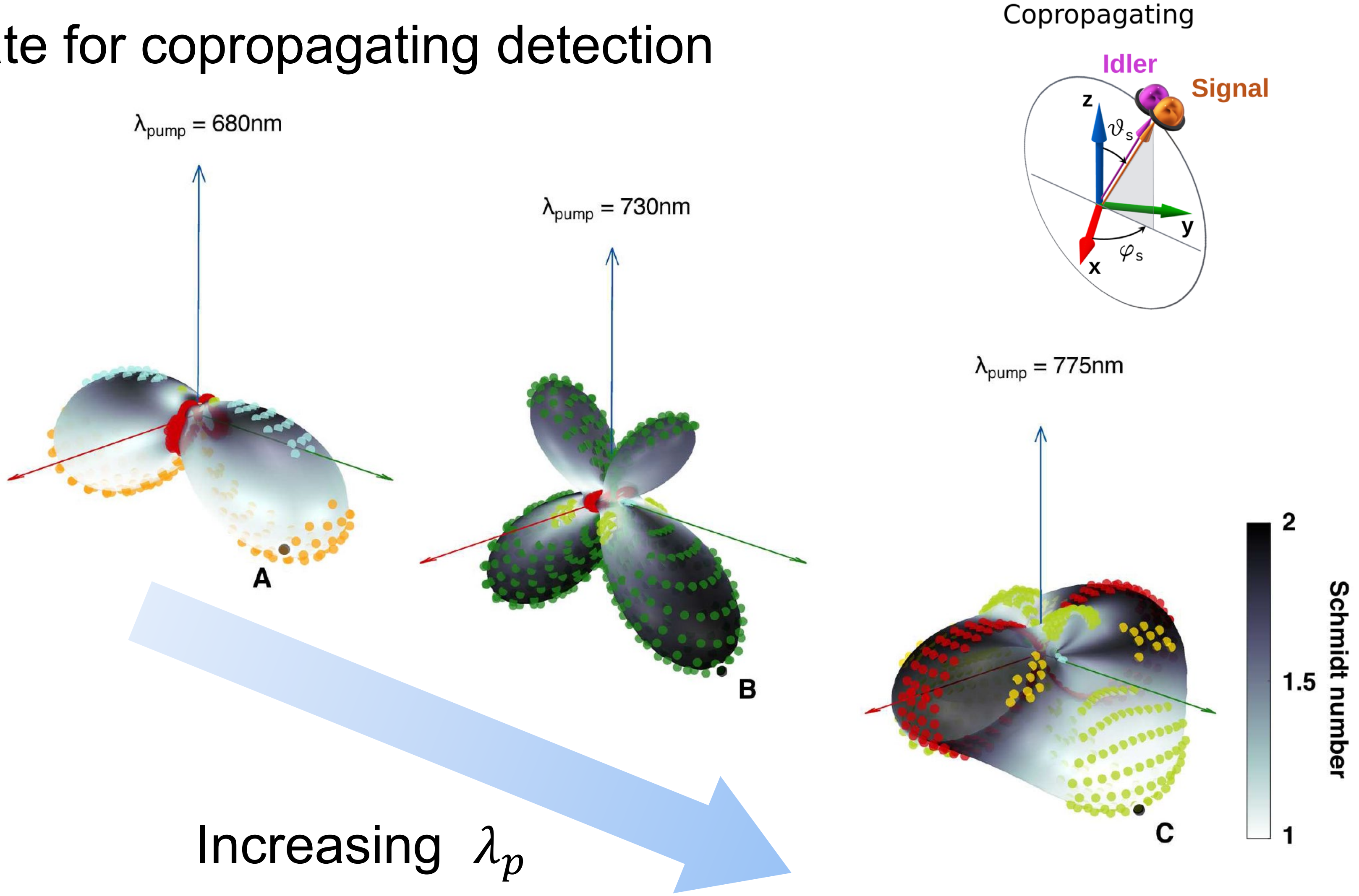
[1]**M. A. Weissflog** *et al.*, „Nonlinear Nanoresonators are Natural Sources of Bell States," Appl. Phys. Rev. 11, 011403 (2024)

Tuning the Polarization Quantum State

Flexible tuning of quantum state for copropagating detection

Change λ_p





Increasing λ_p

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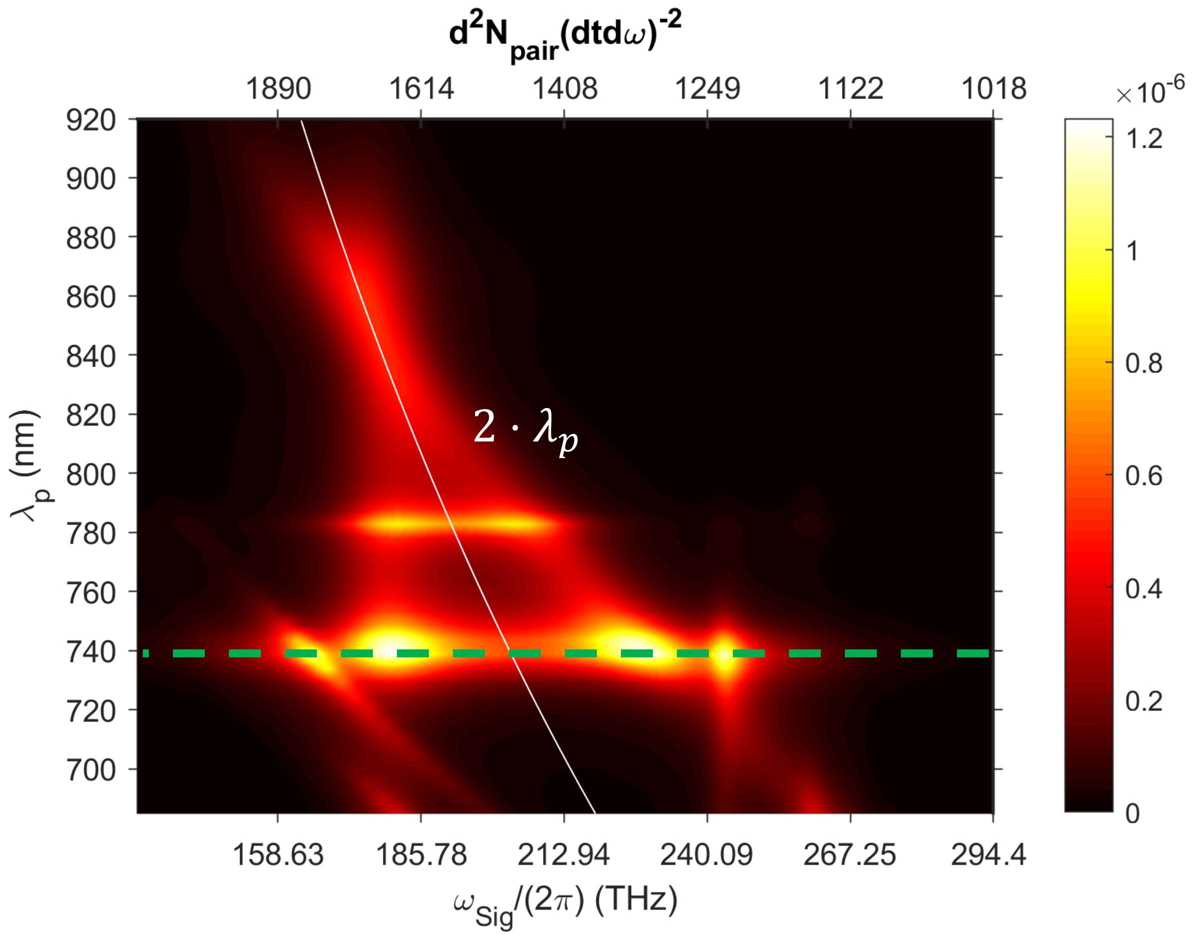
SPDC Spectrum

SPDC spectra for different excitation wavelength (NA=0.8 backwards)



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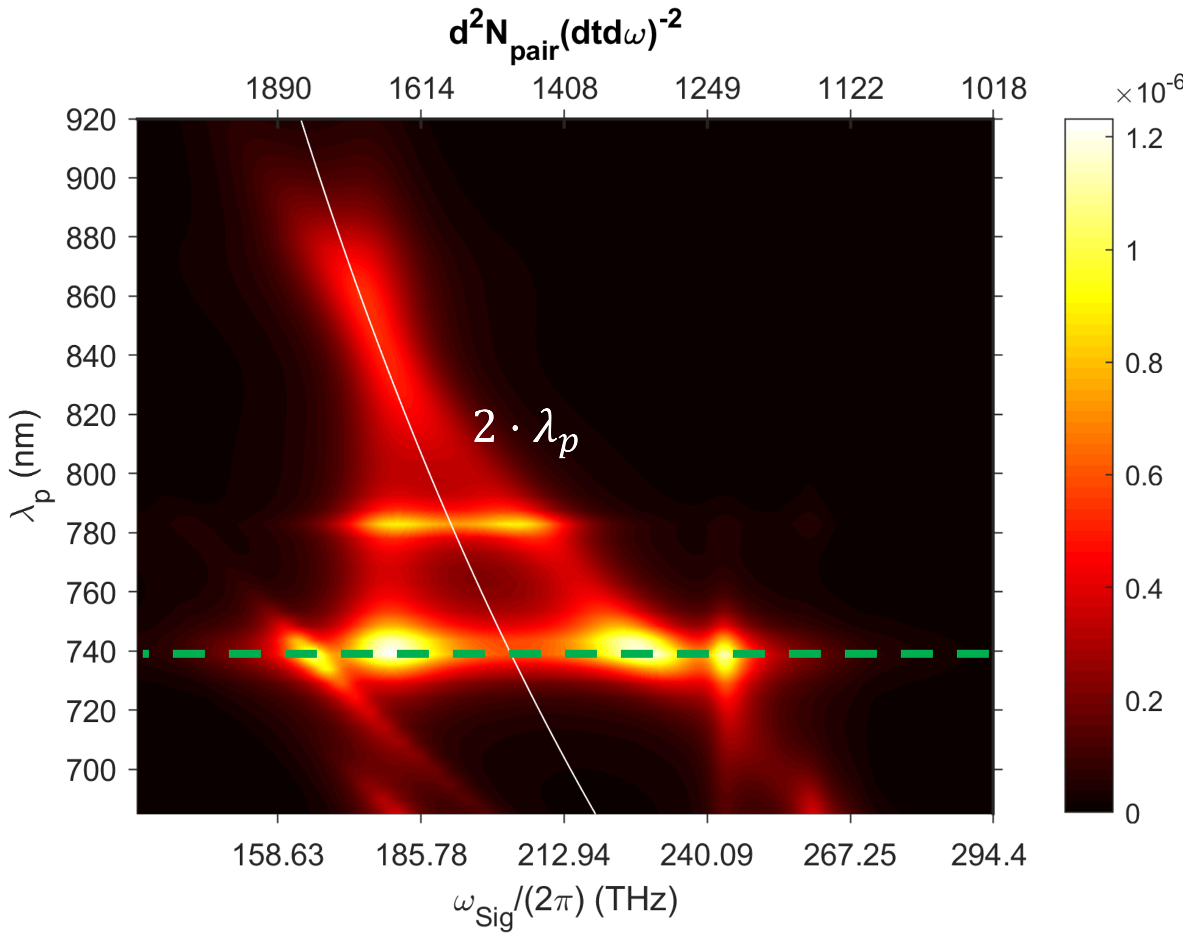
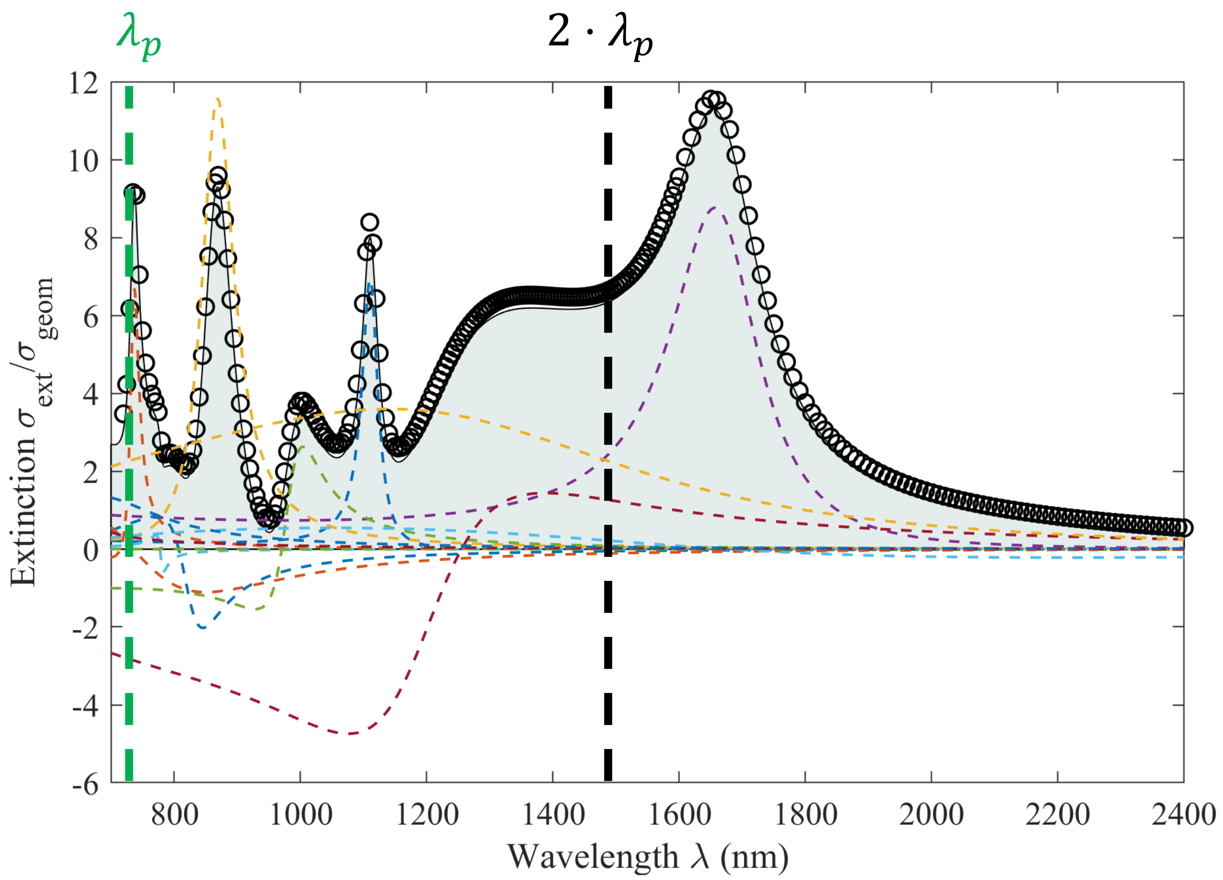
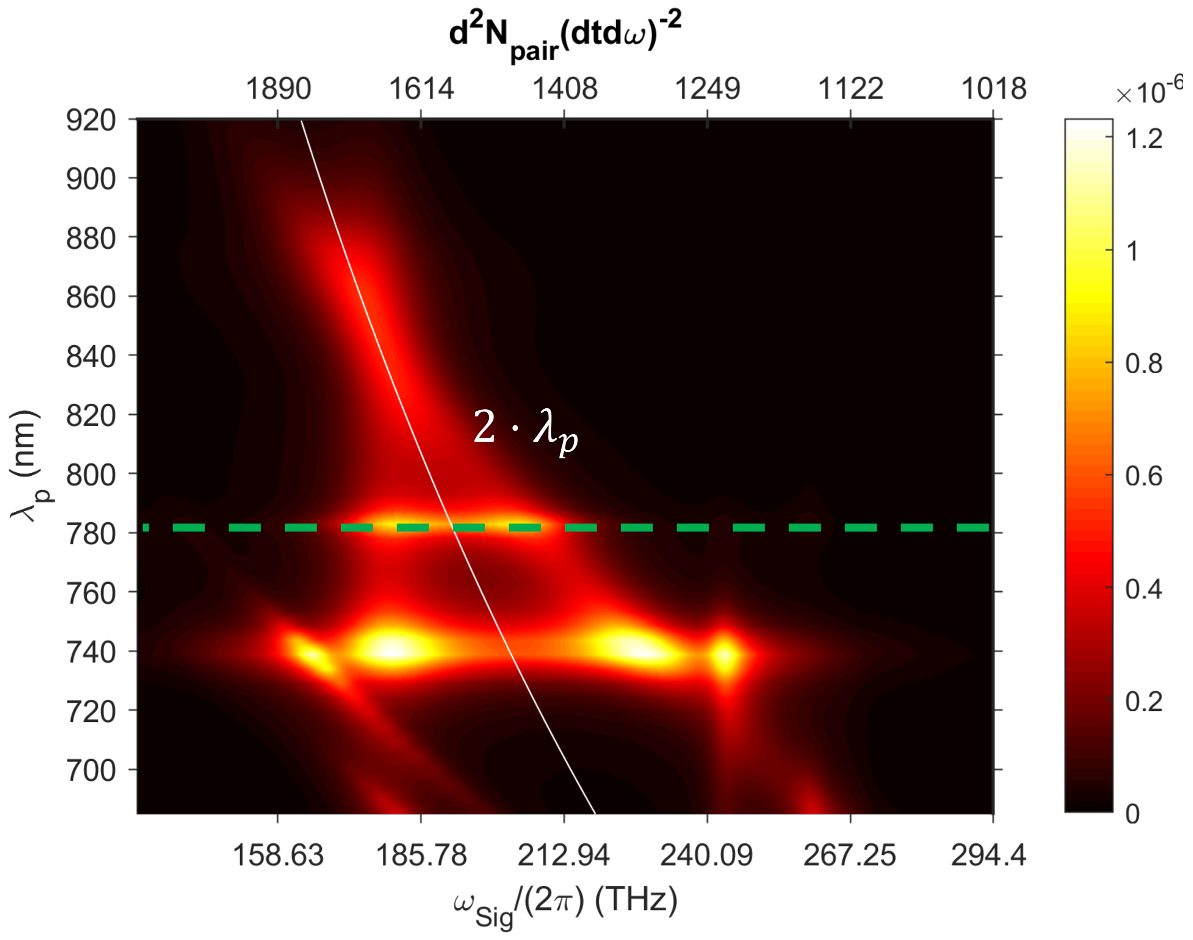


$\lambda_p = 740$ nm

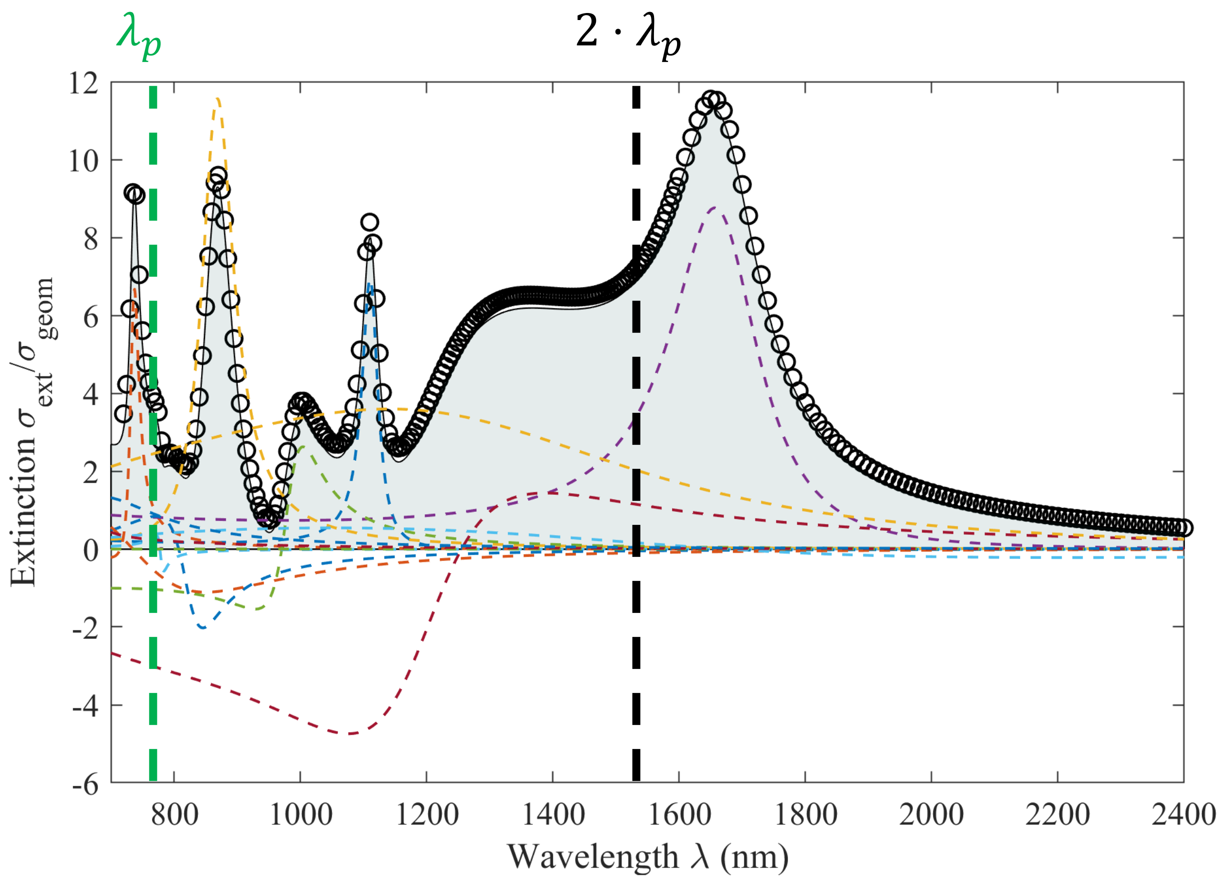


SPDC Spectrum

SPDC spectra for different excitation wavelength (NA=0.8 backwards)



$\lambda_p = 780$ nm



Summary

- Development of model to describe SPDC in nanostructures
- Single nanoresonators can directly generate entangled states
- Different Bell states can be generated